Upper Primary Division

Section A

1. 

[Answer: 11 110]

Solution: Original Expression

 

2. 

[Answer: 2]

Solution: Original Expression 

 

3. 



Solution: Original Expression 

 

4. 



Solution: Original Expression

 

 

5. 



Solution: Original Expression

 

 

 

 

Section B.

1. Johnny used machines to make squares and rectangles. He made each figure using 12 matchsticks (without breaking any matchsticks). Which figure below has not followed the instructions?

 A. B. C. D.

[Answer: C]

Solution:

 Figures A, B, and D use 12 matchsticks. While C uses 14 matchsticks.

2. There are equal numbers of cats, dogs, and chickens in the yard. All in all, there are 50 legs. How many cats are there in the yard?

 A. 4 B. 6 C. 5 D. 7

[Answer: C]

Solution:

 Each cat and dog have 4 legs, each chicken has 2 legs. One cat, one dog and one chicken have 10 legs together, so we have 5 animals of every kind (50 legs ÷ 10 = 5). We know there are equal numbers of cats, dogs and chickens in the yard, so there are 5 cats in the yard.



1. Figure 1 shows the  grids whose area of the shaded part is 37 . What is the area of the unshaded part, in 
2. 43 B. 74 C. 80 D. 111

[Answer: A]

Solution:

 From the given figure, there are 37 small right triangles in the shaded portion, and 80 small right triangles in the entire rectangle.

 

 Then the area of the rectangle is 80  it follows the area of the unshaded portion is 43 

1. A strange animal farm has cats and dogs. The number of dogs is 180 more than the number of cats. However, 20% of the dogs mistakenly think they are cats, and 20% of cats think they are dogs. Among the cats and dogs, 32% of them think they are cats, how many dogs are there in all?

[Answer: A]

Solution:

 Let the number of cats represent as *x* and the number of dogs as *y*.

 From the given information, we set the equation as

 

 Hence, there are  dogs in the farm.

1. Figure 2 consists of the same dimension square and

 the same dimension isosceles right triangle.

 How many squares of different sizes are there?

[**Answer: 83**]

**Solution:**

Let the area of the small grid square in the Figure 2 1 square unit. Sort all the squares according to the size of the area and count:

the number of squares whose area is 1 square units, we have

 

the number of squares whose area is  square units, we have

 

the number of squares whose area is  square units, we have 

the number of squares whose area is  square units, we have 

the number of largest square in the outer frame, we have 1.

Thus, the total number of squares is 

Section C.

1. The Figure 3 shows five isosceles triangles with top angles 24o, 48o, 72o, 96o and 120o the first multiples of the smallest top angle. All top angles have an integer number of degrees. We want to make a similar picture with as many non-overlapping triangles as possible. How many degrees is the smallest top angle in that case?



 Figure 3

Solution:

 If we have *n* triangles, we need 360 to be a multiple of

 1 + 2 + ⋅⋅⋅ + *n*.

 Therefore, 360 must be a multiple of or 720 must be a multiple of *n*(*n* + 1).

 This means that 720 must be a multiple of both *n* and (*n* + 1). We write down the list of factors of 720:

 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 36, 40, 45, 48, 60, 72, 80, 90, 120, 144, 180, 240, 360, 720.

 We find that the largest possible value of *n* is 5.

 Then since  the smallest top angle is 

2. On 22 cards there have been written positive integers from 1 to 22. With these cards 11 fractions have been made. What is the greatest number of these fractions that can have integer values?

Solution:

 Note that not all the 11 fractions can be integers, since there are three prime numbers (13, 17 and 19) which will only produce an integer when paired with 1. However, we can get 10 fractions to be integers:



 Therefore, the answer is 10.

3. The Zoo manager wants to distribute bananas to three groups of monkeys. If he wants to distribute to the first group of monkeys only, then each monkey of that group will get 12 bananas. If distribute to the second group of monkeys only, then each monkey of that group will get 15 bananas. If distribute to the third group of monkeys only, then each monkey of that group will get 20 bananas. When distribute to all the three groups of monkeys, each monkey will get how many bananas?

[Answer: 5]

Solution:

 Total Number of Bananas  the number of monkeys in the first group

  the number of monkeys in the second group

  the number of monkeys in the third group

 Hence, the total number of bananas must be multiple of 12, 15 and 20.

 The least number of bananas that meets the criteria is 60. So the total number of bananas is 
It follows the possible number of monkeys in the first group are  the possible number of monkeys in the second group are  while the possible number of monkeys third group are 

 Therefore, when the bananas are equally divided into three groups of monkeys, the number of bananas per monkey is 5.

4. Seven consecutive non-zero positive integers are respectively placed at the intersections *A*, *B*, *C*, *D*, *E*, *F*, *G* of the five circles in figure below such that the sum of the numbers on each circle is equal. If the number filled in is greater than 0 and not greater than 10, what is the number filled in point *G*?



[**Answer: 7**]

**Solution:**

 Use the same letter to represent the number filled in at each point. From the given information, we have

 

 Thus, the total of all the sum of integers on five circles is twice the sum of the seven consecutive integers. Therefore, the sum of these seven consecutive integers must be divisible by 5. Since 7 cannot be divisible by 5, and the sum of 5 consecutive non-zero positive integers must be divisible by 5, it follows the sum of the smallest and largest integers must be divisible by 5. So,  Among the 7 consecutive positive integers not greater than 10, only the seven numbers of 2, 3, 4, 5, 6, 7, 8 satisfy the requirements. Thus, the sum of the numbers on each circle is 14, so G=7.



5. The seven positive integers 39, 41, 44, 45, 47, 52, 55 are rearranged in a line such that the sum of any three adjacent integers is a multiple of 3. What is the largest possible value of the fourth integer in all such arrangements?

[**Answer: 68**]

**Solution:**

 It is known that after the arrangement of the seven positive integers 39, 41, 44, 45, 47, 52, 55, the sum of any three adjacent integers is required to be a multiple of three. Let us divide the 7 integers by 3, the remainder is 0, 2, 2, 0, 2, 1, and 1. After the corresponding arrangement, since the 7 remainders have only 2 zeros, the 3 adjacent integer are greater than 0 or impossible to be equal to 6, otherwise, the three adjacent remainders are 2, and among the 7 remainders we have three of them are 2. In this case, the sum of the first 3 adjacent remainders must be 4 or 5. Hence, the sum of the remainders of three adjacent integers must be 3.

Since after the seven remainders are arranged, each of the three adjacent remainders is added, then we have 5 times of additions established. In these 5 operations, the occurrences of the remainder is 0, 1, 2 is *x*, *y*, *z*; respectively, so we have

 

So, we obtain

  

It can be obtained that *y* must be an odd number greater than one.

We know that among 7 remainders, 2 must appear 3 times, so that  and  Hence, the solutions of  are

1.  (2)  e can i

In the solution (1), we can interpret the remainder 0 appear 6 times. in the arrangement of 7 remainders, the remainder 0 can only appear in the third, fourth and fifth position. The sum of remainder in 4th, 5th and 6th position is either 1 or 2, which is contradict to the condition of the given problem!

In the solution (2), the position of 1st, 4th and 7th can only be the remainder 2. It indicate the remainder of the 3 positive integers in the 1st, 4th and 7th position can only be an integer divided by 3 by 2.

Thus, according to the requirement of the problem, after the arrangement of 7 positive integers 39, 41, 44, 45, 47, 52, 55, the largest possible integer is the 4th position must be 47.